Exploring Mathematical Decision-Making Through EEG Analysis

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ABSTRACT

In this study, mathematical decision-making tasks were used to provide further details on the flow of information across a number of brain regions, with the objective of finding out whether connectivity patterns are informative in predicting decisional outcomes. The experiment consisted of showing 50 mathematical expressions to each participant, and they decided on their correctness by pressing buttons. Neural activity and button presses were recorded by means of the g.tec Nautilus EEG device, equipped with 64 electrodes. A detailed epochs analysis was conducted with regard to participant responses. Advanced techniques of signal analysis were applied, including Granger causality, Phase Locking Value, and Complex Pearson Correlation Coefficient. This research aims to determine how the following tools could distinguish events from states, get aware of their limitations, and develop novel analysis techniques for better discrimination of brain processes. This research is specifically focused on using mathematical reasoning as a model to study decision-making processes. Our objective is to test existing and develop novel methods for gaining deeper understanding of the brain dynamics involved in discrete cognitive activities.

KEYWORDS

EEG, mathematical decision-making, neural connectivity, connectivity analysis, neural signal classification, EEGNet

1 INTRODUCTION

1.1 Background

Electroencephalography (EEG) devices are core tools in neuroscience for the monitoring of brain activity through the detection of electrical potentials in different places on the scalp [4]. They find applications in a wide variety of both clinical and research settings during the investigation of brain activity and connectivity. The ability to understand the brain processes is crucial for advancements in neuroscience, medical diagnostics and brain-computer interfaces. Identification and improvement of methods that are capable of classifying events and explaining the underlying decision process is also of a great importance.

In this study we aim to set the whole pipeline for conducting such research, which consists of data acquisition and analysis. For our brain process of interest we selected mathematical reasoning, which exemplifies decision-making processes. It is selected because of its complexity that enables layered analysis of sub processes, as mathematical thinking involves complex cognitive processes that engage multiple brain regions. Mathematical decision-making tasks require the integration of numerical processing, working memory, and logical reasoning, making them an ideal for studying brain connectivity.

1.2 Objectives

Our primary objective with this research is to assess existing techniques for connectivity analysis and to develop a comprehensive pipeline for the analysis of brain processes (during mathematical decision-making tasks), with the focus on creation and refinement of methodologies that would be able to classify and explain these processes. Through the comprehensive analysis we also aim to validate two specific hypotheses.

- **H1** - Mathematical thinking causes unique connectivity patterns, differentiable from resting state brain activity.
- **H2** - True and false answers can be distinguished by their EEG signals.

The motivation behind this study is to contribute to the understanding of cognitive processes by providing insights into the neural dynamics of decision-making.

2 LITERATURE REVIEW

EEG has established itself as an invaluable tool in cognitive neuroscience, particularly for exploring brain activity in real time. Its application in understanding the neural mechanisms of mathematical decision-making has drawn considerable interest due to the dynamic and complex nature of the task. Previous studies have demonstrated that specific brain regions, particularly within the frontal and parietal lobes, are significantly active during mathematical cognition, reflecting the intricate process of problem-solving and numerical reasoning [1, 2].

Basic methods for EEG analysis rely on statistical analysis of independent electrode signals, and do not enable reliable differentiation of complex brain activities. This can be achieved by additionally considering the mutual signal interdependence as a reflection of utilization of brain networks, known as brain connectivity analysis. There are several accepted brain connectivity analysis methods, which include Phase Locking Value (PLV), Weighted Phase Locking Index (wPLI), Complex Pearson Correlation Coefficient (CPCC) [7] and Granger Causality (GC). Most methods including PLV, wPLI and CPCC are not directional and rely on analyzing phase differences between the electrode signals. GC is a directional method, developed by Clive Granger in the 1960s, and determines whether one time series can help predicting another one. Applied on EEG data it can reveal directional influences between different neural regions covered by corresponding EEG electrodes.

Granger causality has been widely used in neuroscience to explore the temporal dynamics of brain activity. For example, it has

been applied to EEG signals to investigate the functional connectivity between different brain areas during various cognitive states. Recent research by Seth, Barrett, and Barnett (2015) [6] has further demonstrated the effectiveness of GC in identifying directed functional interactions in neuroscience and neuroimaging time-series data. Their findings indicate that GC can reveal insights into the functional circuits involved in perception, cognition, and behavior. This research also emphasizes both the theoretical foundations and practical applications of GC while discussing its limitations and computational strategies, thus solidifying its role as a crucial tool in neuroscience.

With the advances in artificial intelligence the use of artificial neural networks also affects EEG analysis. One of the most promising artificial neural network architectures for classification of EEG data is EEGNet [3], a compact convolutional neural network designed for EEG-based brain-computer interfaces. Recently, it has been shown that neural networks can contribute to understanding of underlying processes, by computing saliency maps [5]. As such, artificial neural networks could also be extended and utilized to reveal connectivity patterns.

3 METHODOLOGY

3.1 Participants

For the purpose of the study, we recruited 15 participants from the university. Each participant provided written informed consent before participating in this experiment. This work was approved by the university's ethical committee to ensure the study conformed to ethical standards for studies involving human participants.

3.2 Equipment

- EEG Headset: g.tec Nautilus EEG device with 64 channel electrodes.
- Base Station: Connected to the EEG headset for data transmission.
- Trigger Box: Connected to the base station, equipped with two response buttons.
- Optical Sensor: Connected to the trigger box to detect changes in the visual stimuli.
- Recording Software: g.recorder for capturing and storing EEG data.

3.3 Procedure

The experiment was set in the following way:

A participant was seated comfortably in a noise-free, dimly lit room to help eliminate other external factors that might cause discomfort. An EEG headset was fitted on the head of the participant, making sure that the contact of all the electrodes on the scalp is good to ensure high-quality signals. The headset was then connected to the base station and trigger box.

The experiment consisted of 50 mathematical equations that were shown for 10 seconds each on a computer screen, where the research participants had to determine whether the equation was correct or incorrect. Responses were marked by pressing one of the two corresponding buttons connected to the trigger box. At the end of each equation, there was a resting phase of 3 seconds where the subjects could rest before the next equation appears.

An optical sensor was used to exactly capture the display time of each equation, thus ensuring correct synchronization with the visual presentation and EEG data. This setup allowed for exact timing of participant responses relative to the presentation of the equations.

In this experiment we recorded EEG data using the g.recorder software, that captured the continuous EEG signals on all activities of 64 channels at a sampling rate of 250 Hz. The software also recorded the presentation timings of the stimuli and participant responses according to the detection of the optical sensor and trigger box. This setup ensured that all relevant events will be accurately time-stamped and synchronised with the EEG data.

Each of the participants completed the experiment in an individual session, which in total lasted approximately 15 minutes. The data was stored safely for later preprocessing and analysis.

3.4 Data Collection

Figure 1 shows the raw EEG data recorded from one of the subjects while performing the mathematical decision-making task. EEG signals, captured from 64 channels, are shown here along with their respective labels on the y-axis, which represent electrode positions at different locations on the scalp, and the x-axis represents time in seconds.

In this visualization, specific triggers are marked to indicate important events during the experiment. The green line represents a trigger at the moment the equation on screen changes, thus signaling the beginning of a new arithmetic problem. This trigger is important in synchronizing EEG data with the exact moment each equation is presented to the participant, thus providing the ability to analyze the neural response to the stimulus very precisely.

The red line marks the trigger corresponding to the event of a user pressing the "incorrect" button, thus signaling his/her decision that the equation presented is wrong.

Although not shown in the current image, a blue line is used as a trigger to mark the event when a participant presses the "correct" button, indicating their decision that the equation is correct.

3.5 Data Preprocessing

One of the most important steps in satisfying the quality and reliability of the recorded signal before detailed analysis is the preprocessing of EEG data. MATLAB with the EEGLAB toolbox was used for this purpose, where advanced functionalities were applied to deal and handle with the intricate nature of EEG data. The preprocessing pipeline started by filtering all frequencies of the raw EEG data outside the frequency range of interest. That was easily accomplished with the help of a bandpass filter with the limiting frequencies of 0.1 Hz and 45 Hz. This filtering step was quite important to avoid noise or other effects due to muscle activity, electrical interference, etc.

After filtering, the EEG data was re-referenced to the common average reference. This involved averaging the signal from all electrodes and subtracting this average from each individual electrode's signal to give clarity to the signal and remove common noise. Common average referencing is conducted as a standard operation in preprocessing when carrying out EEG. This operation helped to normalize data across different channels.
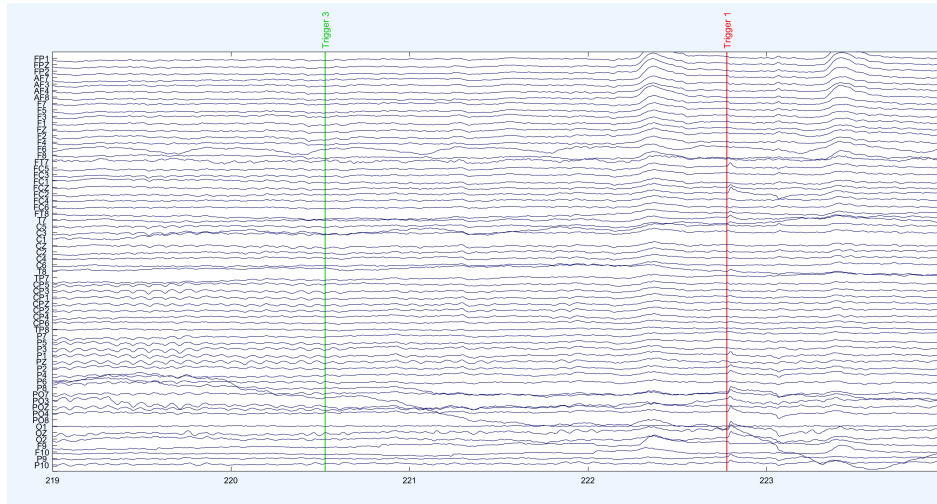
Figure 1: Visualisation of the raw signals data.

Artifacts were removed by Independent Component Analysis (ICA), where the EEG signal was decomposed into independent components. With the help of ICA and the ICLabel add-on in MATLAB, components related to common artifacts due to eye movements, blinks, and muscle activity were identified and isolated. Removing these artifact components from the data ensured that the remaining signals are more representative of the true neural activity.

3.6 Hypothesis Testing

The basic idea of our hypothesis testing approach revolves around developing and training classification methods on epochs that we define specifically around key events (equation change, incorrect/correct marking). For example, for testing the first hypothesis **H1**, two primary states can be defined:

- **Rest** - Epochs taken from a 3-second window just before a new equation appears - shown by this green line in our recording setup. This is the period not active for making any judgment, which in turn gives our baseline or rest state.
- **Active** - In contrast active state epochs taken from a 3-second window just prior to the participants' responses since these active states are thought to carry neural signatures related to the cognitive processes of judgment and decision making on the mathematical expressions.

In this way we can directly compare neural activity in both "decision-making" and "resting" conditions, with the specific objective of the identification of distinct patterns that could validate our hypotheses about the differential brain connectivity in different cognitive states.

For the second hypothesis, **H2**, testing adapts methodologies developed for H1 but focus on epochs particularly related to the correctness of the participant's response. It is hypothesized that EEG signals could differentiate between true and false answers of participants in mathematical decision-making tasks. The epochs

were extracted in a similar way with 3 second intervals before the blue and red triggers in the dataset.

3.7 Connectivity Matrices

Connectivity matrices serve as a fundamental tool in neuroscience for visualizing and quantifying the intricate patterns of neural interactions within the brain. These matrices can be derived with the use of the different connectivity analysis techniques mentioned above.

In Figure 2 we can see the Granger causality matrices obtained from one of the subjects' EEG recordings in resting and active cognitive states respectively. Each matrix describes the directional influences between pairs of EEG electrodes over the scalp. The x-axis labels denote the influencing electrodes, and the y-axis labels indicate the influenced electrodes. Each cell in this matrix thus corresponds to a pair of electrodes; the color of each cell reflects the strength of causal influence from the electrode on the x-axis to the electrode on the y-axis.

The color scale, ranging from 0 to 0.18, is provided at the right side of the matrices. The colors change from cool colors like blue, indicating very weak causal influence, to warm colors like yellow, representing very strong influences. This scale will help the eye in assessing the strength and distribution of connectivity across the brain.

4 RESULTS

After segmenting the epochs and preparing the EEG data from all participants, we used the EEGNet neural network to classify resting and active states, in order to validate **H1**. The network was trained with 80% of the data and tested with the remaining 20%.

This resulted in a classification accuracy of about 84%, showing that distinct neural connectivity patterns are present during mathematical decision-making tasks compared to resting states.
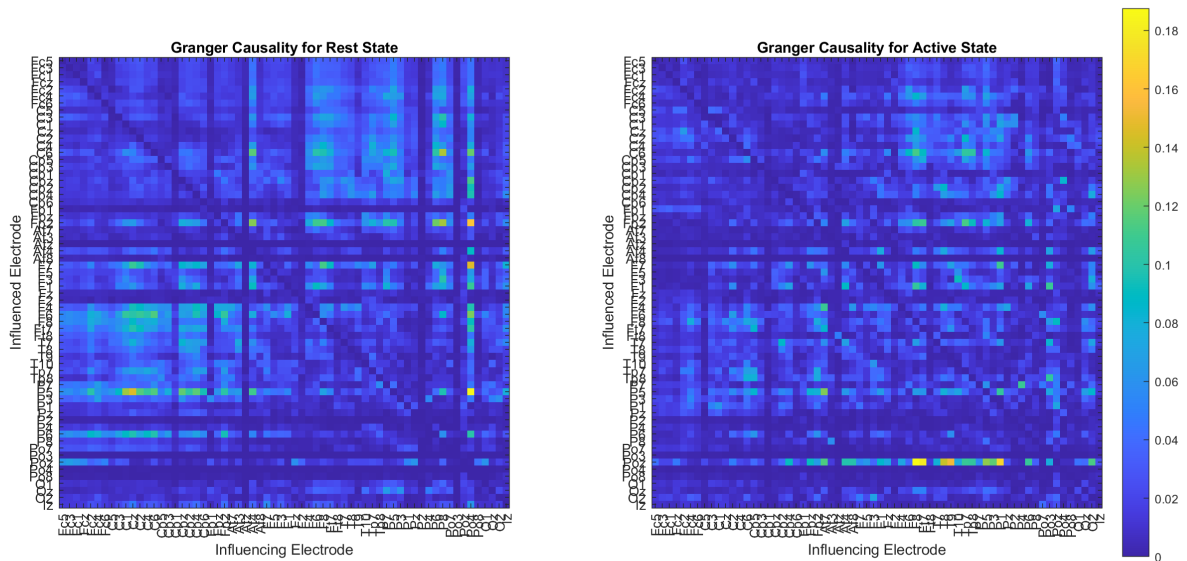
Figure 2: Visualisation of the connectivity matrices of rest and active states.

The findings support our hypothesis that, mathematical thinking causes unique connectivity patterns, differentiable from resting state brain activity. This suggests the promising capability of the EEGNet to discriminate between rest and active states based on the neural data collected around the event-defined epochs.

We also did some testing on the second hypothesis **H2**. Initial tests using the EEGNet neural network for epochs related to correct and incorrect responses resulted in classification accuracies of about 50%, which is clearly insufficient. These results suggest two possible explanations: either the EEG signals do not contain enough distinguishing information, or the applied methods, are not yet optimized to detect subtle differences in brain activity.

Given these results, we will continue to refine our analytical methods and to explore alternative models for a better representation of neural dynamics. Hypothesis **H1** has proven to be a more accessible goal, while hypothesis **H2** presents a greater challenge. Should we confirm **H2**, it could revolutionize how we estimate knowledge and decision-making processes based on neural data.

5 CONCLUSION

The main objective of this research is the development and enhancement of methodologies to analyse EEG signals during cognitive tasks, with a special emphasis on mathematical decision-making. The strategy taken in this research provides a model for future works on more complex cognitive phenomena. It indicates the need for precise acquisition of data, sophisticated preprocessing strategies, and new analytical techniques in an attempt to capture and interpret correctly the activity in the brain.

In summary, this research adds a great deal into the field of development of methodologies that further improve our understanding

of cognitive processes and pushes the boundaries of how we can interact with technology using Brain Computer Interfaces (BCI) and analyse neurological conditions. Further evolution of these methods is likely to close the gap between human cognitive functions and machine interpretation, setting the stage for possible future advances that may change neurological healthcare and technology interfacing.

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